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Matrices Over a Discrete Valuation Ring

by ANTONIO PIZARRO[12]

The goal of this research is to establish canonical forms and similarity classes of matrices with entries in a valuation ring. As a further step, the objective is to use the similarity classes to calculate the representations of the linear group $\mathrm{Gl}(n)$ over the ring of integers in a p-field.

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The case of 2×2 matrices has been solved [2]. The case of 3×3 matrices has been partially solved. Canonical forms and the similarity classes are known [3]. There are some results in the case of $n \times n$ matrices, $n > 2$.

Let R be a ring. Denote by $M_{r \times s}(R)$ the set of $r \times s$ matrices with entries in R. If R is a complete (with the p-adic topology) local Noetherian ring with maximal ideal P, then two matrices are similar over R if and only if they are similar over the Artinian rings R/P^n, $n > 1$. Canonical forms are known for matrices over R/P (since R/P is a field), namely the rational canonical form or a direct sum of rational canonical forms. The problem is to determine under what condition a matrix A over R/P will preserve the same canonical form when viewed as a matrix over R/P^n, $n > 1$. We have shown that cyclic matrices over R/P will remain cyclic over R/P^n, $n > 1$ [3].

Direct Sum of Matrices and Matrix Congruencies

IN 1952 W. E. Roth used the matrix equation $AX - XB = C$ to determine when a block upper triangular matrix in the form

$$\begin{pmatrix} A & B \\ O & C \end{pmatrix}$$

is a direct sum of matrices. The result was extended to matrices over arbitrary rings by W. H. Gustafson [1]. A similar result exists for a full matrix with entries in a complete discrete valuation ring. The proof of the following theorem has been published elsewhere [4].

THEOREM. *Let R be a complete discrete valuation ring with maximal ideal P generated by a prime p. Let $A \in M_{m \times m}(R/P^n)$ and $D \in M_{r \times r}(R/P^n)$ be fixed, and $n > 1$. For $B \in M_{m \times r}(R/P^n)$, $C \in M_{r \times m}(R/P^n)$, $B \not\equiv O \pmod{p}$, $C \not\equiv O \pmod{p}$ define*

$$\mu(B,C) = \begin{pmatrix} A & p^{n-1}B \\ p^{n-1}C & D \end{pmatrix}$$

and denote

$$\mu(O,O) = \begin{pmatrix} A & O \\ O & D \end{pmatrix}.$$

Then for $\mu(B,C)$ to be similar to $\mu(O,O) \pmod{p^n}$ it is necessary and sufficient that the congruences $B \equiv AX - XD \pmod{p}$ and $C \equiv DY - YA \pmod{p}$ have solutions.

The Similarity Classes for 3 × 3 Matrices

We consider the case of 3×3 matrices with entries in R/P^n, $n > 1$, modulo scalar matrices. Every matrix has a cyclic vector, i.e., it is similar to the rational canonical forms, unless the matrix is similar to either

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \pmod p \quad \text{for} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \pmod p$$

It has been shown [3] that in this case the matrices have a canonical form known as essentially cyclic. The form is either

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & p^k \\ a & b & c \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 & p^r & 0 \\ 0 & 0 & 1 \\ a & b & c \end{pmatrix}$$

for k, $r > 1$ and $a, b, c \in R/P^n$. Notice that the characteristic polynomial of the essentially cyclic matrix is congruent to $x^3 \pmod p$. The current stage of our research is considering $n \times n$ matrices, $n > 1$, for which the characteristic polynomial is congruent to $x^n \pmod p$. Our conjecture is that if the characteristic polynomial is irreducible (mod p^n), the canonical form is essentially a cyclic matrix where the ones parallel to the main diagonal are replaced by certain powers of the prime p. In other words, the matrix is similar to

$$\begin{pmatrix} 0 & p^{k_1} & 0 & 0 & \cdots & 0 \\ 0 & 0 & p^{k_2} & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & p^{k_{n-1}} \\ a_1 & a_2 & a_3 & a_4 & \cdots & a_n \end{pmatrix},$$

$k_1, k_2, \ldots$ nonnegative integers, $a_1, a_2, \ldots \in R/P^n$. The conjecture awaits a proof or a counterexample.

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